Foundation / Higher

Maths GCSE Problem Solving Questions Workbook

Indices

GRADES 6 – 9

MATHS-SCHOOL

Laws of indices

EXAMPLE

Find the positive integers for c and d, where c and d are less than 5, such that :

a) $16^c = 2^d$ $\quad \mathbf{16 = 2^4} \quad \boldsymbol{c = 1} \quad \boldsymbol{d = 4}$

b) $5^c = 25^d$ $\quad \mathbf{25 = 5^2} \quad \boldsymbol{c = 2} \quad \boldsymbol{d = 1}$

c) $27^c = 3^d$ $\quad \mathbf{27 = 3^3} \quad \boldsymbol{c = 1} \quad \boldsymbol{d = 3}$

1 Find the lowest positive integers for c and d, where c and d are greater than 1, such that:

a) $8^c = 2^d$ b) $5^c = 25^d$ c) $2^c = 4^d$

2 Find the value of x which make the following equations accurate:

a) $8^x = 64$ b) $x^4 = 4^2$ c) $3^{2x} = 729$

3 Find the lowest positive integers for c and d, where c and d are greater than 1, such that:

a) $2^{x+4} = 4^{x+1}$ b) $8^3 = 2^{2x+1}$ c) $3^{2x} = 9^{4x-6}$

4 Find the value of *a* in each of the following equations:

a) $16\sqrt{2} = 2^a$ b) $27^2 = 3^{2a}$ c) $5^2 = 5\sqrt{a}$

5 Find the value of x in each of the following equations:

a) $3^{2x} \times \frac{1}{9} \times 3^9 = 3^x$ b) $2^{\frac{1}{2}} \times 2^{-2} = 8^a$ c) $5^{3x} \times 125 \div \frac{1}{5} = 5^x$

Indices – Negative and fractions

EXAMPLE

Write as powers of 2

(i) $\frac{1}{2}$ 2^{-1}

(ii) $\sqrt{8}$ $8^{\frac{1}{2}} = (2^3)^{\frac{1}{2}} = 2^{\frac{3}{2}}$

(iii) $\sqrt[3]{4}$ $4^{\frac{1}{3}} = (2^2)^{\frac{1}{3}} = 2^{\frac{2}{3}}$

1 Write as powers of 2

a) $\frac{1}{4}$ b) $\frac{2}{\sqrt{16}}$ c) $\frac{2}{\sqrt[3]{4}}$

2 Write as powers of 3

a) $\frac{1}{9}$ b) $\frac{3}{\sqrt{27}}$ c) $\frac{9}{\sqrt[3]{3}}$

3 Evaluate

a) $\left(\frac{4}{9}\right)^2$ b) $\left(\frac{3}{5}\right)^{-2}$ c) $\left(\frac{36}{25}\right)^{-\frac{1}{2}}$

4 Evaluate

a) $\left(1\frac{7}{9}\right)^{-\frac{1}{2}}$ b) $\left(1\frac{11}{25}\right)^{\frac{3}{2}}$ c) $\left(3\frac{3}{8}\right)^{-\frac{2}{3}}$

5 Evaluate

a) $\left(1\frac{13}{36}\right)^{\frac{1}{2}}$ b) $\left(2\frac{10}{27}\right)^{-\frac{1}{3}}$ c) $\left(5\frac{1}{16}\right)^{\frac{3}{4}}$

Indices – Solving equations

Solve the equations

(i) $10^p = 0.1$ $\quad 0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1} \quad p = -1$

(ii) $(25k^2)^{\frac{1}{2}} = 15$ $\quad \sqrt{25k^2} = 5k \quad 5k = 15 \quad k = 3$

(iii) $t^{-\frac{1}{3}} = \frac{1}{2}$ $\quad t^{-\frac{1}{3}} = \frac{1}{\frac{1}{t^{\frac{1}{3}}}} = \frac{1}{2} \quad t^{\frac{1}{3}} = 2 \quad t = 2^3 = 8$

1 Solve the equations

a) $5^p = 0.2$

b) $(8k^3)^{\frac{1}{3}} = 12$

c) $t^{-\frac{1}{2}} = \frac{1}{5}$

2 Solve the equation

$$\frac{8^{x+1}}{2^x} = 16$$

3 Solve the equation

$$\frac{16^x}{2^{x-1}} = 2^{\frac{1}{2}}$$

4 $x = 2^a \quad y = 2^b$

$xy = 32$ and $2xy^2 = 32$

Find the value of **a** and **b**

5 Find an expression for k in terms for $y^{\frac{5}{4}}$

$y = 16 \times 10^{8k}$ where k is an integer.

Where k is an integer. Give your answer in standard form

Solutions

Page 1 – Laws of indices

1. a) $c = 2 \quad d = 6 : 8^2 = 2^6 = 64$

b) $c = 4 \quad d = 2 : 25^2 = 5^4 = 625$

c) $c = 4 \quad d = 2 : 2^4 = 4^2 = 16$

2. a) $x = 2 : \ 8^2 = 64$

b) $x = 2 : \ 4^2 = 2^4 = 16$

c) $x = 3 : \ 3^6 = 729 \qquad 2x = 6$

3. a) $x = 2 : \ 2^{x+4} = 2^{2(x+1)} \qquad x + 4 = 2x + 2$

b) $x = 4 : \ 2^9 = 2^{(2x+1)} \qquad 2x + 1 = 9$

c) $x = 2 : \ 3^{2x} = 3^{2(4x-6)} \qquad 2x = 8x - 12$

4. a) $a = 4\frac{1}{2} : 16 = 2^4, \sqrt{2} = 2^{1/2} \rightarrow \frac{1}{2} + 4 = 4\frac{1}{2}$

b) $a = 3 : \ 27 = 3^3 \rightarrow 27^2 = (3^3)^2 = 3^6$

$\rightarrow 3^6 = 3^{2a} \rightarrow 2a = 6 \rightarrow a = 3$

c) $a = 25 : \ 5^2 \div 5 = \sqrt{a} \rightarrow 5 = \sqrt{a} \rightarrow a = 5^2 = 25$

5. a) $x = -7 : \ 3^{2x} \times 3^{-2} \times 3^9 = 3^{2x+7} = 3^x$

$2x + 7 = x \rightarrow x = -7$

b) $x = -\frac{1}{2} : 2^{-\frac{3}{2}} = (2^3)^a \rightarrow 2^{-\frac{3}{2}} = 2^{3a} \rightarrow 3a = -\frac{3}{2}$

c) $x = -2 : \ 5^{3x} \times 5^3 \div 5^{-1} = 5^{3x+4} = 5^x$

$\rightarrow 3x + 4 = x \rightarrow 2x = -4 \rightarrow x = -2$

Page 2 – Indices (negative and fractions)

1. a) 2^{-2} **b)** $2^{-1} : \ \frac{2}{4} = \frac{1}{2}$

c) $2^{\frac{1}{3}} \ : \ \frac{2}{4^{\frac{1}{3}}} = \frac{2}{2^{\frac{2}{3}}} = 2^{1-\frac{2}{3}}$

2. a) $3^{-2} : \frac{1}{3^2}$ **b)** $3^{-\frac{1}{2}} : \ 3^1 \div (3^3)^{\frac{1}{2}}$

c) $3^{\frac{5}{3}} : \ 3^2 \div 3^{\frac{1}{3}}$

3. a) $\frac{16}{81}$ **b)** $2\frac{7}{9} : \ \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

c) $\frac{5}{6} : \ \left(\frac{25}{36}\right)^{\frac{1}{2}} = \sqrt{\frac{25}{36}}$

4. a) $\frac{3}{4} : \ \left(\frac{16}{9}\right)^{-\frac{1}{2}} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \sqrt{\frac{9}{16}}$

b) $1\frac{91}{125} : \ \left(\frac{36}{25}\right)^{\frac{3}{2}} = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$

c) $\frac{4}{9} : \ \left(\frac{27}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\frac{2}{3}\right)^2$

5. a) $1\frac{1}{6} : \ \left(\frac{49}{36}\right)^{\frac{1}{2}} = \frac{7}{6}$

b) $\frac{3}{4} : \ \left(\frac{64}{27}\right)^{-\frac{1}{3}} = \left(\frac{27}{64}\right)^{\frac{1}{3}} = \frac{3}{4}$

c) $3\frac{3}{8} : \ \left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

Page 3 – Indices (solving equations)

1. a) $p = -1 : \ 0.2 = \frac{1}{5} = \frac{1}{5^1} = 5^{-1}$

b) $k = 6 \quad : \sqrt[3]{8k^3} = 2k \ \rightarrow \ 2k = 12$

c) $t = 25 \ : \ t^{-\frac{1}{2}} = \frac{1}{t^{\frac{1}{2}}} = \frac{1}{5} \ \rightarrow \ t^{\frac{1}{2}} = 5 \ \rightarrow \ t = 5^2$

2. a) $x = \frac{1}{2} : \frac{2^{3(x+1)}}{2^x} = 2^4 \ \rightarrow \ 3x + 3 - x = 4 \rightarrow \ 2x = 1$

3. $x = -\frac{1}{6} : \ \frac{2^{4x}}{2^{x-1}} = 2^{\frac{1}{2}} \quad 4x - (x - 1) = \frac{1}{2}$

$3x + 1 = \frac{1}{2} \ \rightarrow \ 6x + 2 = 1 \ \rightarrow \ 6x = -1$

4. a = 6 , b = −1 : $2^a \times 2^b = 2^5$

index laws → a + b = 5 (1) → $2 \times 2^a \times 2^{2b} = 2^5$

using indices → 1 + a + 2b = 5 → a + 2b = 4(2)

(2) – (1) b = −1 → Since a + b = 5, a = 6

5. $3.2 \times 10^{10k+1} : \ y \ = \ 16 \ \times \ 10^{8k}$

$y^{\frac{5}{4}} \ = \ 16^{\frac{5}{4}} \times (10^{8k})^{\frac{5}{4}} \ \rightarrow \ y^{\frac{5}{4}} \ = 32 \times 10^{10k}$

$y^{\frac{5}{4}} \ = 3.2 \times 10 \times 10^{10k}$