Name:

Prove the following:

a) $(n+2)^2 - (n-2)^2$ is a multiple of 8.

$= (n + 2)(n + 2) - (n - 2)(n - 2)$
$= (n^2 + 4n + 4) - (n^2 - 4n + 4)$
$= 8n$ which is a multiple of 8

b) $(2n+3)^2 - (2n-3)^2$ is divisible by 8

$= (2n + 3)(2n + 3) - (2n - 3)(2n - 3)$
$= (4n^2 + 12n + 9) - (4n^2 - 12n + 9)$
$= 24n = 8(3n)$ which is a multiple of 8

b) $(7n+1)^2 - (7n-1)^2$ is a multiple of 4.

$= (7n + 1)(7n + 1) - (7n - 1)(7n - 1)$
$= (49n^2 + 14n + 1) - (49n^2 - 14n + 1)$
$= 28n = 4(7n)$ which is a multiple of 4

d) The sum of any three consecutive whole numbers is a multiple of 3.

$n + (n + 1) + (n + 2)$
$= 3n + 3$
$= 3(n + 1)$ which is a multiple of 3

e) The sum of any four consecutive numbers is a multiple of 2.

$n + (n + 1) + (n + 2) + (n + 3)$
$= 4n + 6$
$= 2(2n + 3)$ which is a multiple of 2

f) The square of any odd number is also odd.

$(2n + 1)^2$
$= 4n^2 + 4n + 1$
$= 2(2n^2 + 2n) + 1$ which is always odd

g) The product of any two consecutive odd numbers leaves a remainder of three when divided by four.

$(2n + 1)(2n + 3)$
$= 4n^2 + 8n + 3$
$= 4(n^2 + 2n) + 3$ so has a remainder of 3

**Exam question:**
Prove that the difference between the squares of any two consecutive whole numbers is equal to the sum of the same two numbers.

$(n + 1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1$

$n + (n + 1) = 2n + 1$ so always equal